Memory effects and virial corrections in nonequilibrium dense systems

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A dense interacting Bose or Fermi gas is considered. Within the Kadanoff-Baym real-time Green-function technique a generalized kinetic equation is derived in the nonequilibrium ladder approximation. As a Markovian limit the Boltzmann-Uehling-Uhlenbeck equation is recovered. The relevance of retardation effects in the kinetic equation of dense systems is shown. It turns out that the stationary solution of the generalized kinetic equation reproduces the second virial coefficient of the equation of state, including the Beth-Uhlenbeck quantum corrections. We establish a generalized kinetic equation describing correlation effects in a consistent approach to nonequilibrium for Bose and Fermi systems. In this way we give a generalization of the Beth-Uhlenbeck formula that applies to nonequilibrium and to finite systems. The derivation makes use of the time dependent T-matrix equation, which establishes a generalization of the Bethe-Goldstone equation. The correlated parts of the density and energy, which can be interpreted as virial corrections, are thermodynamically consistent. In this way global energy conservation is fulfilled.

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I. INTRODUCTION

The dynamical description of correlated systems in nonequilibrium is a current frequently investigated task. The concerned physical systems range from hot compressed nuclear matter, semiconductor transport to dense nonideal plasmas. For instance, most approaches to particle production properties in heavy ion reactions use the Boltzmann-Uehling-Uhlenbeck (BUU) approach [1–5], in that they assume instantaneous binary collisions neglecting the collision duration time and, therefore, retardation effects.

Other approaches to the kinetic theory of higher densities show the evidence of these effects [6]. Recent calculations establish an effect to the relaxation times in relativistic heavy ion collisions [7] or in plasma situations [8]. One result is that the correlation time of two particles or the collision duration time $\tau_{\rm mem}$ interferes with the mean field relaxation time $\tau_{\rm MF}$.

In conventional microscopic calculations it is assumed that the mean-field relaxation time is small compared with the momentum relaxation time $\tau_{\rm MF} \ll \tau_{\rm rel}$. Therefore, one usually uses a local potential interaction for the mean field, whereas the collisions are performed with a finite range interaction [9]. These collision integrals contain a transition matrix and the statistical Pauli blocking for Fermions. By this way, only the Hartree-Fock energy is conserved. Consequently, conventional transport of the BUU type is only justified if $\tau_{\rm mem} \ll \tau_{\rm MF} \ll \tau_{\rm rel}$.

The success of these calculations in describing one-particle dynamics and experiments is striking over a great energy range [9–13]. Nevertheless some questions remain unsolved, among them should be mentioned: (i) The transverse properties of collective flow are unsatisfactorily described [14]. (ii) The formation of bound states is not included in the solved kinetic equations but added in a second step by a coalescence model [15,16].

What have these two main problems in common, the inclusion of bound states and the retardation effects? A preliminary hint to the close connection can be found from the consideration of the above introduced different time scales. In the case that the system forms bound states during collisions, the collision duration time become very large compared with the relaxation time of the system. The bound state can be considered as a long living correlation in the system. In this sense, memory effects become important if bound states arise.

In the thermodynamical picture, the one-particle density is expected to consist of a free and a correlated part $n = n_f + n_{\rm corr}$, where the correlated part of the density has a contribution from the bound states and from the scattering states of two correlated particles. Until now, this problem was concerned in equilibrium approaches (see [17] and citations therein). In equilibrium, the bound state contributions to the equation of state is described by the Beth-Uhlenbeck approach [18]. The generalized Beth-Uhlenbeck approach goes beyond the Bruckner-Bethe-Goldstone theory. It reflects the fact that correlated nucleons can form bound states and scattering states. By this way, it is possible to describe the Mott transition as pressure ionization in a many-particle framework [17]. A further interesting feature is the occurrence of two phase transitions, i.e., the liquid gas phase transition and the phase transition to the superfluid state. Similar investigations are performed for nonideal plasmas and for excitons in semiconductors [19]. The simultaneous treatment of bound state formation and Bose condensation is one of the challenging problems, which can be studied by this approach.

The kinetic theory for such strongly correlated systems and, therefore, the proper nonequilibrium approach is much more complicated. Several attempts are made to generalize the kinetic equations to dense quantum systems [20–27]. This description of correlated particles leads to virial corrections to the equation of states

[21–23]. The correct Beth-Uhlenbeck virial correction was given, e.g., in [28]. The improvement of the quantum kinetic equations are still in discussion [29]. Currently, Snider gives a generalized kinetic equation [30], which leads to a reinterpretation of the distribution function in correlated and free parts [31]. Other approaches incorporate the correlations into the equation of motion in a stochastic approximation [32]. This results in a Boltzmann-Langevin equation for the fluctuating single-particle density [33].

The aim of the present paper is to close the gap between a kinetic approach including memory and the description of correlated states. It will be shown that the first order retardation effects in the kinetic equation leads to the same expression for the correlated density or the energy in equilibrium, which is known from the quantum Beth-Uhlenbeck approach. We can link the approaches of equilibrium to the kinetic level of description and generalize it by this way to nonequilibrium situations found in finite systems. Therefore, we consider this as a generalization of the Beth-Uhlenbeck approach to nonequilibrium and finite systems.

The general starting point in deriving kinetic equations is the coupled set of equations of motion for the reduced density operators. This set was derived by Irving and Zwanzig [34]. The formal structure is similar to the so-called Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy [35] for reduced distribution functions in classical statistical physics. The first to use this BBGKY hierarchy in deriving kinetic equations were Bogoljubov, Born and Green, and Kirkwood [36–38]. The quantum Boltzmann equation differs from the classical one in the collision term, which takes into account that the final scattering states can be occupied and, consequently, blocked by the Pauli exclusion principle for Fermionic systems. Moreover, the quantum mechanical transition rate is used, rather than the classical one. Of the various other extensions proposed for the ordinary Boltzmann equation, the work of Klimontovich and Kremp [39], McLennan [40], and Röpke and Schulz [41] should be mentioned, which treat quantum gases with bound states, for a review see [42,43].

A fundamental assumption in deriving kinetic equations, Bogoliubov's principle of weakening of initial correlations, was generalized by Zubarev [44]. By this way it is possible to treat also long living correlations leading to the the Enskog equation.

The most powerful and elegant techniques to describe the entire region of density and temperature as well as situations far from equilibrium, where linear response fails, is the method of the quantum-statistical Green's function. It was introduced in 1955 by Matsubara [45] for describing many-particle systems. In 1959, Martin and Schwinger [46] introduced Green's-function techniques for the description of correlated many-body systems starting from a hierarchy of equations of motion for the imaginary-time Green's functions. Kadanoff and Baym [47] presented a complete transport theory based on these many-body techniques in 1962. An important component in the construction of this transport theory was an analytic continuation of the equilibrium Green's function from the imaginary times to real times. This step was necessary to describe nonequilibrium situations.

At about the same time, Schwinger [48] initiated the use of real-time Green's functions. These functions are defined on a directed contour and are sometimes also called path-ordered Green's functions. Both Keldysh [49] and Craig [50] used this concept and formulated, with the help of perturbative methods, a Dyson equation. The real-time Green's functions allow the properties of many-particle systems to be investigated in a ground state, in a finite temperature equilibrium state and in a nonequilibrium situation, in a consistent manner.

Any subsequent studies merely provided an alternative derivation either of the kinetic equation itself or of the T-matrix equation, the most elaborate of which was given by Danielewicz [6]. This derivation mostly based itself on a simple approximation by means of a perturbation series. However, in doing so, it was not clear what this approximation implied for higher-order Green's functions. This method was extended by Kremp and co-workers [25,51] and later on, it was used by Danielewicz [52] as well as Botermans and Malfliet [42].

Applications of real-time many-body methods range from problems of quantum chromodynamics [53], through nuclear physics [42,54–56], to theory of liquid helium [57], physics of plasmas [58], physics of condensed matter [59], astrophysics [60], and cosmology [61]. Thus, it is the most general formalism of the many-particle theory we have today.

The outline of the paper is as follows. In Sec. II, we give a short derivation of the Kadanoff-Baym equation with the special emphasis on the equivalence of an appropriate initial condition and the Keldysh contour. This leads to the quantum kinetic equation with exact time behavior for the one-particle distribution function. This equation is not closed, which is a problem that is handled by the spectral properties of a system. We will discuss this point with a certain ansatz, exact at the level of Hartree-Fock self-energies. The resulting quantum kinetic equation shows retardation.

In Sec. III, we derive a generalized Bethe-Goldstone equation including nonequilibrium Pauli blocking and retardation. With the help of this general T-matrix approximation for the self-energy, we end with the generalized kinetic equation in Sec. IV. From this equation, it is shown in Sec. V that in first order retardation the virial corrections follow, which are equivalent to the Beth-Uhlenbeck terms. As demonstrated in Sec. VI, the corrections to the density and to the energy are thermodynamically consistent also in the case of quasiparticle energies, which are density and temperature dependent.

II. REAL-TIME GREEN'S-FUNCTION TECHNIQUE

A. Exact kinetic equations

We consider a system of interacting Fermions or Bosons with the Hamiltonian,

$$H = \frac{1}{V_0} \sum_1 E_1 \, a_1^\dagger \, a_1 + \frac{1}{2V_0^4} \sum_{1,2,1',2'} \langle 12|V|1'2'\rangle a_{1'}^\dagger a_{2'}^\dagger a_2 a_1,$$

(1)

with $1 = (r, s, \tau, \ldots,)$ denoting the single-particle variables (orbital, spin, isospin, etc), V_0 is the normalization volume. The annihilation and creation operators for Fermions or Bosons are, respectively,

$$[a_1, a_2]_\pm = 0, \qquad [a_1, a_2^\dagger]_\pm = \delta_{12},$$

where upper signs refer to the Fermions and lower signs to the Bosons.

In order to describe correlations in highly nonequilibrium situations, we define various correlation functions by different products of creation and annihilation operators in the Heisenberg picture,

$$G^>(1,2) = \frac{1}{i}\langle a_1(t_1)a_2^\dagger(t_2)\rangle,$$

$$G^<(1,2) = \mp\frac{1}{i}\langle a_2^\dagger(t_2)a_1(t_1)\rangle.$$

(2)

Here the $\langle \rangle$ denotes the average value with the unknown statistical *nonequilibrium* operator ρ. The causal Green function is given by

$$G(1,2) = \Theta(t_1 - t_2)G^>(1,2) - \Theta(t_2 - t_1)G^<(1,2).$$

(3)

It is furthermore useful to introduce the retarded and advanced Green functions according to

$$G^R(1,2) = \Theta(t_1 - t_2)\left[G^>(1,2) - G^<(1,2)\right],$$

$$G^A(1,2) = \Theta(t_2 - t_1)\left[G^<(1,2) - G^>(1,2)\right].$$

(4)

Instead of determining the nonequilibrium statistical operator, we follow another concept of statistical mechanics and use the equation of motion of the creation and annihilation operators to derive kinetic equations, which may be solved with the appropriate choice of boundary and initial conditions. Applying the equation of motion for the field operators in the Heisenberg picture one finds the Martin-Schwinger hierarchy [47], where the single-particle Green's function couples to the two-particle one, etc. A formally closed equation for the single-particle Green's function can be reached with the introduction of the self-energy,

$$\mp \, i \int d2V(1-2)G_2(12,1'2^+) = \int_C d\bar{1}\Sigma(1,\bar{1})G_1(\bar{1},1'),$$

(5)

where the contour of integration C has to be determined in such a way that the boundary conditions are fulfilled.

The self-energy can be split into two parts,

$$\Sigma(1, 1') = \Sigma_{\mathrm{HF}}(1, 1') + \Sigma_c(1, 1'),$$

(6)

with the Hartree-Fock part,

$$\Sigma_{\mathrm{HF}}(1, 1') = \mp i\frac{1}{V_0^2} \sum_{22'}[V(121'2')$$

$$\mp V(122'1')]G(22')_{t_2=t_2'}^<,$$

(7)

and the remaining correlation part, which can be written in analogy to the causal Green function,

$$\Sigma_c(1, 2) = \Theta(t_1 - t_2)\Sigma^>(1, 2) - \Theta(t_2 - t_1)\Sigma^<(1, 2).$$

(8)

In order to obtain solutions and the path of integration C, it is necessary to specify the initial conditions. In many physical situations, the condition of weakening of initial correlation is an appropriate choice [25],

$$\lim_{t_1 \to -\infty} G_2(121'2')|_{t_1 = t_2 + \epsilon, \, t_{1'} = t_{2'} + \epsilon}$$

$$= G(11')G(22') \mp G(12')G(21').$$

(9)

From this equation, we get

$$\int_C d\bar{1}\Sigma(1,\bar{1})G(\bar{1},1') = \int_{-\infty}^{+\infty} d\bar{1}\,\{\Sigma(1,\bar{1})G(\bar{1},1')$$

$$-\Sigma^<(1,\bar{1})G^>(\bar{1},1')\}.$$

(10)

It is easy to see that the boundary condition is fulfilled, since the contribution (10) vanishes in the limit $t_1' = t_1^\pm \to -\infty$. For the case $t_1 < t_1'$, we can write, e.g.,

$$\int_{-\infty}^{+\infty} d\bar{t}_1\,\{\Sigma(1,\bar{1})G(\bar{1},1') - \Sigma^<(1,\bar{1})G^>(\bar{1},1')\}$$

$$= \int_{-\infty}^{t_1} d\bar{t}_1\Sigma^>(1,\bar{1})G^<(\bar{1},1') + \int_{t_1}^{t_1'} d\bar{t}_1\Sigma^<(1,\bar{1})G^<(\bar{1},1')$$

$$+ \int_{t_1'}^{\infty} d\bar{t}_1\Sigma^<(1,\bar{1})G^>(\bar{1},1')$$

$$- \int_{-\infty}^{\infty} d\bar{t}_1\Sigma^<(1,\bar{1})G^>(\bar{1},1') \to 0.$$

(11)

Equation (11) can be split into two parts [25], according to

$$\int_{-\infty}^{+\infty} d\bar{t}_1 = \int_{-\infty}^{t_1'} d\bar{t}_1 + \int_{t_1'}^{-\infty} d\bar{t}_1$$

and a contour of time integration follows, which is equal to the Keldysh contour.

With the expressions (5) and (10), we can finally write the first equation of the Martin-Schwinger hierarchy in the following form

$$\left(i\hbar \frac{\partial}{\partial t_1} + \frac{(\frac{\hbar}{i}\nabla_1)^2}{2m} \right) G_1(1,1')$$

$$= \delta(1-1') \mp i \int_C d2 V(1-2) G_2(121'2^+)$$

$$= \int_C d\bar{1}\, \Sigma(1,\bar{1}) G(\bar{1},1')$$

$$= \int_{-\infty}^{+\infty} d\bar{1}\, \left\{ \Sigma(1,\bar{1}) G(\bar{1},1') - \Sigma^<(1,\bar{1}) G^>(\bar{1},1') \right\}.$$

$$(12)$$

Using the definition (3), the Kadanoff-Baym equation, first derived by Kadanoff and Baym and Keldysh [47,49], is obtained in the following manner

$$\left(i\hbar \frac{\partial}{\partial t_1} + \frac{(\frac{\hbar}{i}\nabla_1)^2}{2m} \right) G_1^{\gtrless}(1,1')$$

$$= \int d\bar{r}_1 \Sigma^{HF}(r_1,\bar{r}_1,t_1) G_1^{\gtrless}(\bar{r}_1,t_{1'}1')\, \delta(t_1 - t_1')$$

$$+ \int_{-\infty}^{t_1} d\bar{1}\, \left[\Sigma^>(1,\bar{1}) - \Sigma^<(1,\bar{1}) \right] G_1^{\gtrless}(\bar{1},1')$$

$$- \int_{-\infty}^{t_1'} d\bar{1}\, \Sigma^{\gtrless}(1,\bar{1}) \left[G_1^>(\bar{1},1') - G_1^<(\bar{1},1') \right]. \qquad (13)$$

Subtracting the adjoined equation and setting $t_1 = t_1'$, we finally obtain the time diagonal part [62], i.e., the equation for the Wigner distribution function $f_a^W(pRT) = \mp i G_a^<(pRT\tau = 0)$ of species a,

$$i\left[\hbar \frac{\partial}{\partial T} + \left(\frac{p}{m_a} + \frac{\partial \Sigma_a^{HF}(pRT)}{\partial p} \right) \frac{\partial}{\partial R} - \frac{\partial \Sigma_a^{HF}(pRT)}{\partial R} \frac{\partial}{\partial p} \right] f_a^W(pRT)$$

$$= \int_{-\infty}^{0} d\tau \left[\left\{ G_a^> \left(p,R,T-\frac{\tau}{2},\tau \right), \Sigma_a^< \left(p,R,T-\frac{\tau}{2},-\tau \right) \right\}_+ - \left\{ G_a^< \left(p,R,T-\frac{\tau}{2},\tau \right), \Sigma_a^> \left(p,R,T-\frac{\tau}{2},-\tau \right) \right\}_+ \right].$$

$$(14)$$

Here, we have introduced Wigner coordinates following $\tau = t - \bar{t}$ and $T = (t + \bar{t})/2$. Further, p is the Fourier-transform of $r = r_1 - r_1'$ and the macroscopic space variable is $R = (r_1 + r_1')/2$. This equation is exact in time and we used gradient expansion for space variables assuming small orbital density fluctuations compared with the system anisotropy [62]. Nonlocal terms can be derived by expanding up to a second order in space gradients [21].

Another compactly written form of the Kadanoff Baym equation is given in (A4). Using the quasiparticle approximation in (A4) and neglecting the time retardation in the correlation functions, one derives the well-known Landau-Silin equation. This equation yields the Landau, Boltzmann or Lennard-Balescu equation depending on the approximation one chooses for the self-energy. The different self-energy renormalization terms at the drift side in (A4) are condensed in the incomplete time integration of (14).

This shows the close relation between the self-energy correlation terms in (A4) and the memory in (14). This will be made explicit in Sec. IV. Therefore, we focus on the retardation effects and keep the time convolution exact.

B. The problem of ansatz

In order to close the kinetic equation (14), it is necessary to know the relation between $G^>$ and $G^<$. This problem is known as ansatz and must be constructed self-consistently with the required approximation of the self-energy. The conventional way is to change the correlation

functions to the generalized distribution function and to the spectral one in frequency domain, which is an exact transformation,

$$G^< = \mp ia(p\omega RT) F(p\omega RT),$$
$$G^> = ia(p\omega RT)[1 - F(p\omega RT)]. \qquad (15)$$

Neglecting the off-pole part in the spectral function, one finds a relation of the correlation function and the Wigner distribution function f^W. One gets for $G^<$ in time space,

$$G^<(p\tau RT) = \mp i e^{-\frac{i}{\hbar}[\epsilon(pRT)\tau]} f_W(pRT), \qquad (16)$$

with ϵ the quasiparticle energy (26). This is quite good as long as the quasiparticle picture holds true and no memory effects play a role. This is connected with the neglect of the correlated parts in the equation of state (27). Therefore the simple ansatz, denoted above as *conventional ansatz*, will certainly fail in dense correlated systems. This will be shown explicitly when the equivalence of retardation effects and the correlation part in (25) will be shown in Sec. V.

Another obscure discrepancy is the fact that with the conventional ansatz, we have some minor differences in the resulting collision integrals, if we compare it with the results from the density operator technique [62,63]. With the conventional ansatz, we get just one half of all retardation times in the various time arguments [62,64]. This annoying difference has remained obscure until the recent work of Lipavsky et al. [65]. The general construction of this generalized ansatz is given in the Appendix B. Here,

we give the derivation for the Hartree-Fock approximation, where the ansatz is exact. Using the time diagonal Hartree-Fock self-energy in the equation for the retarded Green function (II C), one can show that the semigroup relation holds [65],

$$iG^R(tt_1) \cdot G^R(t_1t_2) = G^R(tt_2). \tag{17}$$

Now the integral form of the Kadanoff Baym equations (13) reads [6]

$$G^<(11') = G^R(12) \left[(G_0^R(23))^{-1} G_0^<(34)(G_0^A(45))^{-1} \right.$$

$$\left. + \Sigma^<(25) \right] G^A(51').$$

Here, we concentrate only on the time evolution, 1 means t_1, and assume summation over equal indices. First let us set $t_1 > t_1'$. Then by using the property (17), one finds

$$G^<(11') = iG^R(11') \cdot G^<(1'1').$$

Including the opposite case $t_1' > t_1$, one derives

$$G^<(11') = iG^R(12)G^<(22) - iG^a(12) \cdot G^<(11)$$

$$= \mp i\, f_W\left(p, T - \frac{|\tau|}{2}\right) a(p, \tau, T), \tag{18}$$

and analogous

$$G^>(11') = i\left[1 \mp f_W\left(p, T - \frac{|\tau|}{2}\right)\right] a(p, \tau, T). \tag{19}$$

Therefore, the connection between $G^>$ and $G^<$ is given and the kinetic equation can be closed. If we neglect the time retardation in the distribution functions, we recover the conventional ansatz (16).

This ansatz is superior to the conventional Kadanoff-Baym ansatz in several respects: (i) it has the correct spectral properties, (ii) it preserves causality, (iii) the quantum kinetic equations derived with Eq. (A4) coincide with those obtained with the density matrix technique [62], and (iv) it reproduces the Debye-Onsager relaxation effect [66].

In order to get more physical insight into this ansatz, one can transform to frequency representation,

$$g^<(k\omega RT) = \mp 2\, i \int_0^\infty d\tau \cos\left(\frac{1}{\hbar}\left[\hbar\omega\tau - \epsilon(k, R, T)\tau\right]\right)$$

$$\times f\left(k, T - \frac{\tau}{2}\right). \tag{20}$$

Neglecting the retardation in f, one recovers the conventional ansatz (16). By this way, the generalized ansatz provides causality [65].

C. Spectral properties

The knowledge of the retarded Green function provides the spectral function and, therefore, the spectral proper-

ties of the system. The Dyson equation for the retarded one reads

$$\left(i\hbar\frac{\partial}{\partial t_1} + \frac{(\frac{\hbar}{i}\nabla_1)^2}{2m}\right) G^R(1, 1')$$

$$= \delta(1 - 1') + \int d\bar{1}\, \Sigma^R(1, \bar{1}) G^R(\bar{1}, 1'). \tag{21}$$

From this equation, it is now possible to derive an exact expression for the inverse retarded Green's function. First, we see from (II C) the following form of the inverse functions, which may be understood as operators in time space when the internal integration is performed,

$$(G^R)^{-1}(rR\tau T) = -\left[i\hbar\frac{1}{2}\frac{\partial}{\partial T} - i\hbar\frac{\partial}{\partial \tau}\right.$$

$$\left. + \frac{(\frac{\hbar}{i}\frac{1}{2}\nabla_R - \frac{\hbar}{i}\nabla_r)^2}{2m}\right]\delta(\tau)\delta(r)$$

$$- \Sigma^R(rR\tau T). \tag{22}$$

If we add the adjoined equation and use the properties of δ functions, we find in the momentum-frequency representation the exact expression for the inverse Green function,

$$(G^R)^{-1}(pR\omega T) = \left[\omega - \frac{p^2}{2m}\right] - \Sigma^R(pR\omega T). \tag{23}$$

The problem is to find the retarded Green function itself for any approximation of Σ^R, see [67,68].

With conventional gradient expansion, we can invert the field-free Dyson equation [69] (II C) up to *a second order* gradient expansion,

$$a(p\omega RT)$$

$$= \frac{2\,\mathrm{Im}\Sigma^R(p\omega RT)}{[\omega - \frac{p^2}{2m} - \mathrm{Re}\Sigma^R(p\omega RT)]^2 + [\mathrm{Im}\Sigma^R(p\omega RT)]^2}. \tag{24}$$

For small imaginary parts of the self-energy and, consequently, a small damping, one can expand this expression following [70,19]

$$a(p\omega RT) = 2\pi\delta(\omega - \epsilon(pRT))$$

$$\times \left[1 + \frac{\partial}{\partial E}\mathrm{P}\int \frac{d\bar\omega}{\pi}\frac{\mathrm{Im}\Sigma^R(p\bar\omega RT)}{\bar\omega - \epsilon(pRT)}\right]$$

$$- \frac{\partial}{\partial \omega}\mathrm{P}\frac{1}{\omega - \epsilon(pRT)}\mathrm{Im}\Sigma^R(p\omega RT), \tag{25}$$

where P denotes the principal value. The quasiparticle energies $\omega = \epsilon(pRT)$ are a solution of the dispersion relation,

$$\omega - \frac{p^2}{2m} - \mathrm{Re}\Sigma^R(p\omega RT) = 0. \tag{26}$$

It is noteworthy to remark that (25) fulfills the spectral sum rule [71],

$$\int \frac{d\omega}{2\pi} a(p\omega RT) = 1.$$

The total one-particle density follows from the spectral function multiplied with the distribution function,

$$n_a[RT] = \frac{1}{V_0} \sum_p f_W^a(pRT)$$

$$-\frac{1}{V_0} \sum_p \int \frac{d\omega}{2\pi} \mathrm{Im}\Sigma_a(p\omega RT)[F(\omega pRT)$$

$$-f_W^a(pRT)]\frac{\partial}{\partial\omega}\mathrm{P}\frac{1}{\omega - \epsilon_a(pRT)}. \qquad (27)$$

The first one contains the contribution of the freely moving particles, whereas the second one absorbs the contribution of the correlated pairs. This procedure leads to the generalized Beth-Uhlenbeck approach [17]. In equilibrium situations, the resulting thermodynamic properties was discussed in several papers [17,19].

III. T-MATRIX APPROXIMATION

In order to describe short-ranged two-particle interactions, it is necessary to introduce the standard approximation of the many-particle theory, the T-matrix approximation [6,52,47]. In the following, we derive this approximation with full influence of the time dependence, assuming that the Kadanoff-Baym equations are valid (13). This establishes a generalization of the known results.

From Eq. (5), the introduced self-energy can be obtained from the two-particle Green's function by

$$\Sigma_a(1,1') = \mp i \sum_b \int d2d1''d3d3' V_{ab}(1233')$$

$$\times G_2^{ab}(33'1''2^+)G_a^{-1}(1''1'), \qquad (28)$$

where the existence of the inverse Green's function is ensured by the weakening of initial correlation as discussed in the preceding chapter. Further, we introduced the different sorts of particles by a, b, which may be distinguishable by spin, isospin, etc.

In order to consider only binary collision approximation in the two-particle Green's functions, we can write for the causal one [25],

$$G_2^{ab}(121'2^+) = G^a(11')G^b(22^+) \mp \delta_{ab}G^a(12^+)G^b(21')$$

$$+i \int d\bar{1}d\bar{1}'d\bar{2}d\bar{2}'$$

$$\times \left[G^a(1\bar{1})G^b(2\bar{2}) \mp \delta_{ab}G^a(1\bar{2})G^b(2\bar{1})\right]$$

$$\times \langle\bar{1}\bar{2}|T_{ab}|\bar{1}'\bar{2}'\rangle G^a(\bar{1}'1')G^b(\bar{2}'2^+). \qquad (29)$$

From (28), we can now easily conclude that the causal self-energy becomes

$$\Sigma_a(11') = \mp i \sum_b \int d2d2' G^a(2'2^+) \left[\langle12|T_{ab}|2'1'\rangle\right.$$

$$\left.\pm\delta_{ab}\langle12|T_{ab}|1'2'\rangle\right]. \qquad (30)$$

Here, the sum of ladder diagrams is defined as causal T matrix,

$$\langle12|T_{ab}|1'2'\rangle = V_{ab}(121'2') + i \int d\bar{1}d\bar{2}d3d3' V_{ab}(1233')G^a(3\bar{1})G^b(3'\bar{2})\langle\bar{1}\bar{2}|T_{ab}|1'2'\rangle$$

$$= V_{ab}(121'2') + i \int d\bar{1}d\bar{2}d3d3' \langle12|T_{ab}|\bar{1}\bar{2}\rangle G^a(\bar{1}3)G^b(\bar{2}3')V(33'1'2'). \qquad (31)$$

Since the interacting potential is assumed to be local in time, we can simplify the general equations. First we get, from the definition of the T matrix (31),

$$\langle12|T|1'2'\rangle = \langle x_1x_2t_1|T|x_1'x_2't_1'\rangle\delta(t_1 - t_2)\delta(t_1' - t_2')$$

and, therefore, for the causal self-energy

$$\Sigma_a(11') = \mp i \sum_b \int dx_2dx_2' G^a(x_2't_1'x_2t_1^+)$$

$$\times\langle x_1x_2t_1|T_{ab}^{ex}|x_1'x_2't_1'\rangle. \qquad (32)$$

Here and in the following, T^{ex} shall indicate the (anti-)symmetrized T matrix corresponding to (30). From the consideration of the Langreth-Wilkins rules (Appendix A), we can immediately take the relations for the other Green's functions, which can be seen directly

from (32). To save space, we write the equation in operator notation and assume all integration and variables to be the same as in (32),

$$\Sigma^< = \mp i G^> T_{ex}^<$$

$$\Sigma^{R/A} = \mp i T_{ex}^{R/A}G^< \pm i T_{ex}^< G^{R/A}$$

$$= \mp i G^< T_{ex}^{R/A} \pm i G^{R/A}T_{ex}^<. \qquad (33)$$

The assumption of time local potentials results in interesting relations for the defining equation of the T matrix (31). With the abbreviation

$$\langle x_1x_2t|\mathcal{G}^{ab}|\bar{x}_1\bar{x}_2\bar{t}\rangle = G^a(x_1t\bar{x}_1\bar{t})G^b(x_2t\bar{x}_2\bar{t}), \qquad (34)$$

we get the causal T matrix,

$$\langle x_1 x_2 t | T_{ab} | x_1' x_2' t' \rangle = V_{ab}(x_1 - x_2, x_1' - x_2') \delta(t - t')\, 2\, \delta(x_1 + x_1' - x_2 - x_2')$$

$$+ i \int d\bar{x}_1 d\bar{x}_2 dx_3 dx_3' V_{ab}(x_1 - x_2, x_3 - x_3') \langle x_3 x_3' t | \mathcal{G}_{ab} | \bar{x}_1 \bar{x}_2 \bar{t} \rangle \langle \bar{x}_1 \bar{x}_2 \bar{t} | T_{ab} | x_1' x_2' t' \rangle . \qquad (35)$$

Now we can again apply the Lengreth-Wilkins rules of Appendix A and find the relations in operator notation

$$T^{\gtrless} = i V \mathcal{G}^R T^{\gtrless} + i V \mathcal{G}^{\gtrless} T^A,$$
$$T^{R/A} = V + i V \mathcal{G}^{R/A} T^{R/A}. \qquad (36)$$

By replacing the operator $1 - iV\mathcal{G}^R$ in the first equation with the help of the second one, we derive the generalized optical theorem [6,42,25]:

$$\langle x_1 x_2 t | T_{ab}^{\gtrless} | x_1' x_2' t' \rangle = \int d\bar{x}_1 d\bar{x}_1' d\bar{x}_2 d\bar{x}_2' d\bar{t} d\bar{t}'$$
$$\times \langle x_1 x_2 t | T_{ab}^R | \bar{x}_1 \bar{x}_2 \bar{t} \rangle$$
$$\times \langle \bar{x}_1 \bar{x}_2 \bar{t} | \mathcal{G}_{ab}^{\gtrless} | \bar{x}_1' \bar{x}_2' \bar{t}' \rangle$$
$$\times \langle \bar{x}_1' \bar{x}_2' \bar{t}' | T_{ab}^A | x_1' x_2' t' \rangle. \qquad (37)$$

Below we will use the Fourier-transformed two-particle functions, which we introduce in the following way:

$$\langle x_1 x_2 | T(tt') | x_1' x_2' \rangle = \int \frac{d\mathbf{P}}{(2\pi\hbar)^3} \frac{d\mathbf{p}}{(2\pi\hbar)^3} \frac{d\mathbf{p}'}{(2\pi\hbar)^3}$$
$$\times e^{i\frac{\mathbf{P}}{\hbar}(x_1 + x_2 - x_1' - x_2')/2}$$
$$\times e^{i\frac{\mathbf{p}}{\hbar}(x_1 - x_2) + i\frac{\mathbf{p}'}{\hbar}(x_1' - x_2')}$$
$$\times \langle \mathbf{p} | T(\mathbf{P} R tt') | \mathbf{p}' \rangle, \qquad (38)$$

where $R = (x_1 + x_2 + x_1' + x_2')/4$. With the help of this notation we get from (32), the following closed set of equations to determine the self-energy in T-matrix approximation, provided the connection between $G^<$ and $G^>$ is known,

$$\Sigma_a^{\gtrless}(pRtt') = \mp i \sum_b \int \frac{d\bar{p}}{(2\pi\hbar)^3} G_b^{\lessgtr}(\bar{p} R t' t)$$
$$\times \left\langle \frac{p - \bar{p}}{2} \left| (T_{\text{ex}})_{ab}^{\gtrless}(p + \bar{p}, R, t, t') \right| \frac{p - \bar{p}}{2} \right\rangle \qquad (39)$$

and

$$\left\langle \frac{p - \bar{p}}{2} \left| T_{ab}^{\gtrless}(p + \bar{p}, R, t, t') \right| \frac{p - \bar{p}}{2} \right\rangle = \int \frac{dp' dp''}{(2\pi\hbar)^6} d\bar{t} d\bar{t}' \left\langle \frac{p - \bar{p}}{2} \left| T_{ab}^R(p + \bar{p}, R, t, \bar{t}) \right| p' \right\rangle \left\langle p' \left| \mathcal{G}_{ab}^{\gtrless}(p + \bar{p}, R, \bar{t}\bar{t}') \right| p'' \right\rangle$$
$$\times \left[\left\langle p'' \left| T_{ab}^A(p + \bar{p}, R, \bar{t}', t') \right| \frac{p - \bar{p}}{2} \right\rangle \mp \delta_{ab} \left\langle p'' \left| T_{ab}^A(p + \bar{p}, R, \bar{t}', t') \right| \frac{\bar{p} - p}{2} \right\rangle \right], \qquad (40)$$

with

$$\langle p' | \mathcal{G}_{ab}^{\gtrless}(p + \bar{p}, R, \bar{t}\bar{t}') | p'' \rangle = (2\pi\hbar)^3 \delta(p' + p'') G_a^{\gtrless}\left(\frac{p + \bar{p}}{2} + p', R, \bar{t}\bar{t}' \right) G_b^{\gtrless}\left(\frac{p + \bar{p}}{2} + p'', R, \bar{t}\bar{t}' \right). \qquad (41)$$

From the T-matrix equations (36), we get for the retarded one

$$\langle p | T_{ab}^R(PRtt') | p' \rangle = V_{ab}(p, p') \delta(t - t') + i \int \frac{d\bar{p}}{(2\pi\hbar)^3} \int_{-\infty}^{t} d\bar{t} V_{ab}(p, \bar{p}) \left[G_a^{>}\left(\frac{P}{2} - \bar{p}, R, t\bar{t} \right) G_b^{>}\left(\frac{P}{2} + \bar{p}, R, t\bar{t} \right) \right.$$
$$\left. - G_a^{<}\left(\frac{P}{2} - \bar{p}, R, t\bar{t} \right) G_b^{<}\left(\frac{P}{2} + \bar{p}, R, t\bar{t} \right) \right] \langle \bar{p} | T_{ab}^R(PR\bar{t}t') | p' \rangle. \qquad (42)$$

This system of equations were derived assuming small microfluctuations in space in comparison to macroobservables R. Thus gradient expansions in space can be applied. It is important to remark that equations (39,40,41) contain the exact time behavior for both, microscopic times as well as center-of-mass times. Taking furthermore into account the Lipavsky ansatz to close the equation for $G^<$ and $G^>$, see (B7), we obtain after changing the variables $x = \bar{\tau} + \tau$:

$$\langle p | T_{ab}^R(K, \tau RT) | p' \rangle = V_{ab}(p, p') \delta(\tau) - i \int \frac{d\bar{p}}{(2\pi\hbar)^3} V_{ab}(p, \bar{p}) \int_0^\tau dx$$
$$\times \left\langle \bar{p} \left| T_{ab}^R\left(K, x, T + \frac{x - \tau}{2}, R \right) \right| p' \right\rangle e^{\frac{i}{\hbar}[\epsilon_a(K/2 - \bar{p}, R, T) + \epsilon_b(K/2 + \bar{p}, R, T)](x - \tau)}$$
$$\times \left[1 - f_a\left(\frac{K}{2} - \bar{p}, T - \frac{\tau}{2} + x, R \right) - f_b\left(\frac{K}{2} + \bar{p}, T - \frac{\tau}{2} + x, R \right) \right]. \qquad (43)$$

To make further progress it is useful to discuss the different times, which occur in the preceding integral. First, we have the difference time τ and x, which couples the scattering events to their statistically weighted values $f(K, T)$. Second,

we see a complicated *non-Markovian* behavior by retardation of center variables, which means that the system has a memory of former events. The x integration additionally couples to a fast oscillating phase itself. Therefore, it is justified to apply a *saddle-point approximation*, which means we have to take the values in retardation times as $x = 0$. Then the resulting equation takes the form

$$\langle p|T_{ab}^R(K, \tau RT)|p'\rangle = V_{ab}(p, p')\delta(\tau) - i \int \frac{d\bar{p}}{(2\pi\hbar)^3} V_{ab}(p, \bar{p}) \int_0^\tau dx$$

$$\times \left\langle \bar{p} \left| T_{ab}^\tau\left(K, x, T - \frac{\tau}{2}, R\right)\right|p'\right\rangle e^{\frac{i}{\hbar}[\epsilon_a(K/2-\bar{p},R,T)+\epsilon_b(K/2+\bar{p},R,T)](x-\tau)}$$

$$\times \left[1 - f_a\left(\frac{K}{2} - \bar{p}, T - \frac{\tau}{2}, R\right) - f_b\left(\frac{K}{2} + \bar{p}, T - \frac{\tau}{2}, R\right)\right]. \tag{44}$$

It is possible to rewrite (44) in a more familiar way

$$\langle p|T_{ab}^R(K, \omega T)|p'\rangle = V_{ab}(p, p') - i \int \frac{d\bar{p}}{(2\pi\hbar)^3} V_{ab}(p, \bar{p}) \int \frac{d\omega}{2\pi} \int \frac{d\bar{\omega}}{2\pi} \frac{e^{i(\bar{\omega}-\omega)\tau}\langle \bar{p}|T_{ab}^R(K, \bar{\omega}, T - \frac{\tau}{2})|p'\rangle}{\epsilon_a(K/2 - \bar{p}, R, T) + \epsilon_b(K/2 + \bar{p}, R, T) - \bar{\omega} - i\epsilon}$$

$$\times \left[1 - f_a\left(\frac{K}{2} - \bar{p}, T - \frac{\tau}{2}, R\right) - f_b\left(\frac{K}{2} + \bar{p}, T - \frac{\tau}{2}, R\right)\right]. \tag{45}$$

The equilibrium limit is the familiar Bethe-Goldstone equation. The equation (44) with the factor $1 \mp f \mp f = (1 \mp f)(1 \mp f) - ff$ is, in principle, more than the Bloch-de Dominicis equation [72] with a respective factor $(1 \mp f)(1 \mp f)$, by allowing for both, intermediate particle-particle and hole-hole, excitations. In the fermion zero-temperature limit, Eq. (44) corresponds to the Galitskii [73] equation, while the Bloch–de Dominicis equation corresponds to the Bethe-Goldstone [74] equation. Differences between the equations have been discussed in detail in the literature (for references see [6]).

Here, we like to point out that (45) is an off-shell equation for the T matrix. Therefore, correlations like bound states are included. This will become crucial for the discussions in the next sections.

For the required self-energies (39), we use the optical theorem (37) to eliminate $T^{\gtrless}$ by retarded functions. Then we can follow each step described above for the T matrix, and using the same saddle-point approximation, we obtain, in time domain the form,

$$\Sigma_a^<\left(k_a, \tau, T - \frac{\tau}{2}\right) = \text{Re} \sum_b \int \frac{d\mathbf{k_a'}d\mathbf{k_b}d\mathbf{k_b'}}{(2\pi\hbar)^6} \delta\left(\mathbf{k_a} + \mathbf{k_b} - \mathbf{k_a'} - \mathbf{k_b'}\right)$$

$$\times [1 - f(k_b)] f(k_a') f(k_b') e^{\frac{i}{\hbar}\{[\epsilon(k_b)-\epsilon(k_a')-\epsilon(k_b')]\tau\}}$$

$$\times \left\langle \frac{k_a - k_b}{2} \left| T_{ab}^R(k_a + k_b, \omega, T)\right| \frac{k_a' - k_b'}{2}\right\rangle$$

$$\times \left\langle \frac{k_a' - k_b'}{2} \left| (T_{ex}^A)_{ab}(k_a + k_b, \omega, T - \tau)\right| \frac{k_a - k_b}{2}\right\rangle\bigg|_{\omega=\epsilon(k_a')+\epsilon(k_b')}, \tag{46}$$

where $f(k_a) = f(k_a, R, T - \tau)$.

IV. KINETIC EQUATIONS

In order to evaluate the general kinetic equation (14), we use the expression for the self-energy in the T-matrix approximation obtained above. The generalized kinetic equation (14), reads, finally,

$$i\left[\frac{\partial}{\partial T} + \left(\frac{k_a}{m_a} + \frac{\partial \Sigma_{\text{HF}}^a(k_a RT)}{\partial k_a}\right)\frac{\partial}{\partial R_a} - \frac{\partial \Sigma_{\text{HF}}^a(k_a RT)}{\partial R_a}\frac{\partial}{\partial k_a}\right] f_W^a(k_a RT)$$

$$= \text{Re} \int \frac{d\mathbf{k_a'}d\mathbf{k_b}d\mathbf{k_b'}}{(2\pi\hbar)^6} \delta\left(\mathbf{k_a} + \mathbf{k_b} - \mathbf{k_a'} - \mathbf{k_b'}\right) \int_0^\infty d\tau e^{\frac{i}{\hbar}[\epsilon(k_a)+\epsilon(k_b)-\epsilon(k_a')-\epsilon(k_b')]\tau}$$

$$\times \{[(1 - f(k_a)][1 - f(k_b)] f(k_a') f(k_b') - [1 - f(k_a')][1 - f(k_b')] f(k_a) f(k_b)\}$$

$$\times \left\langle \frac{k_a - k_b}{2} \left| T_{ab}^R[k_a + k_b, \epsilon(k_a') - \epsilon(k_b'), T]\right| \frac{k_a' - k_b'}{2}\right\rangle \left\langle \frac{k_a' - k_b'}{2} \left| (T_{ex}^A)_{ab}[k_a + k_b, \epsilon(k_a') - \epsilon(k_b'), T - \tau]\right| \frac{k_a - k_b}{2}\right\rangle. \tag{47}$$

Here $f(k) \equiv f(k, T - \tau)$ stands for using the generalized ansatz. Neglecting the retardation in the distribution functions, one obtains the usual quantum Boltzmann equation. If we had used the conventional ansatz, the retardation in the distribution functions would be one half. This is inconsistent with the density operator technique, which may be seen from the Born approximation [62]. The Born approximation of the collision integral in (47) takes the following form:

$$I_{ab} = \frac{2}{\hbar^2} \int \frac{d\mathbf{k}'_a d\mathbf{k_b} d\mathbf{k}'_b}{(2\pi\hbar)^9} \delta(\mathbf{k_a} + \mathbf{k_b} - \mathbf{k}'_a - \mathbf{k}'_b) V^2_{ab}(\mathbf{k_a} - \mathbf{k_a}') \int_0^\infty d\tau \cos[(\epsilon_a + \epsilon_b - \epsilon'_a - \epsilon'_b)\tau/\hbar]$$

$$\times \{f'_a f'_b (1 - f_a)(1 - f_b) - f_a f_b (1 - f'_a)(1 - f'_b)\} \tag{48}$$

where $f(k_a) = f(k_a, R, T - \tau)$.

This equation can be generalized to high field problems [62,67], which was first derived by Levinson [63] for electron phonon interaction by density operator technique. Other authors have redeveloped this result by the Green's-function technique [75] with a $\frac{1}{2}$ discrepancy in the retardation times, which was corrected later by the generalized ansatz. Reference [76] specified the most convenient gauge for the dc field. This field-dependent Boltzmann equation is most commonly used in integral form [77], where it is named Barker-Ferry equation [64]. Similar kinetic equation were derived for a pulse excited semiconductor transport [78–80]. Without the used saddle-point approximation, one retains with double time integrations [81]. There an additional damping is introduced to ensure convergence. It can also be found in the generalization of magnetic fields in [82,83]. Further it is interesting to remark that this equation fulfills the global energy conservation, i.e., the sum of kinetic and correlation energy [8,84].

In order to get more physical insight into the retardation effects, we expand the kinetic equation up to the first order in retardation times. After this we will calculate the retardation completely. From the phase factor of (47), one concludes that the first order expansion is justified for times $t \gg \tau_{\rm mem}$ with the memory or collision duration time $\tau_{\rm mem} \equiv 1/E$.

Choosing as typical energy the Fermi-energy results in the known Landau criterion. Incidentally, in the early 1950's the criterion $\hbar/k_B T < \tau$ was supposed to limit the validity of the Landau Fermi-liquid theory for metals [85]. Later it was shown by Landau that this criterion is irrelevant and he proposed the correct criterion $\tau \approx 1/E_F$, which is much less restrictive in metals. For a semiconductor situation [86] it yields $\tau \approx 10^{-14}$ s, which is negligible compared to ordinary relaxation times around 10^{-10} s. In a typical nuclear situation this collision duration time is around 2 fm/c, where a Fermi energy at nuclear density is used. This can be compared with ordinary relaxation times of 200 fm/c in the same nuclear system to show that this expansion (49) is sufficient.

Now we proceed to prove that this first order retardation effect is equivalent to the Beth-Uhlenbeck correlation density describing bound states. From the spectral representation of the T matrix, one has $T^R T^A = 1/2|T|^2$ neglecting higher order correlations and the first order expansion of the collision integral (47) reads

$$\left[\frac{\partial}{\partial T} + \left(\frac{k_a}{m} + \frac{\partial \Sigma_{\rm HF}(k_a R T)_a}{\partial k_a}\right)\frac{\partial}{\partial R_a}\right.$$

$$\left. - \frac{\partial \Sigma_{\rm HF}(k_a R T)_a}{\partial R_a}\frac{\partial}{\partial k_a}\right] f_a(k_a R T)$$

$$= \sum_b I^B_{ab} + \sum_b I^{\rm ret}_{ab}. \tag{49}$$

I^B_{ab} is the quantum mechanical Boltzmann collision term,

$$I^B_{ab} = \frac{1}{\hbar} \int \frac{d\mathbf{k}'_a d\mathbf{k_b} d\mathbf{k}'_b}{(2\pi\hbar)^6} \delta(\mathbf{k_a} + \mathbf{k_b} - \mathbf{k}'_a - \mathbf{k}'_b)$$

$$\times |\langle k_a k_b | T^R_{ab} | k'_a k'_b \rangle|^2 2\pi \delta(E_{ab} - E'_{ab})$$

$$\times \{f'_a f'_b (1 \mp f_a)(1 \mp f_b) - (1 \mp f'_a)(1 \mp f'_b) f_a f_b\}, \tag{50}$$

whereas $I^{\rm ret}_{ab}$ describes the retardation effect

$$I^{\rm ret}_{ab} = -\frac{\partial}{\partial T} \int \frac{d\mathbf{k}'_a d\mathbf{k_b} d\mathbf{k}'_b}{(2\pi\hbar)^6} \delta(\mathbf{k_a} + \mathbf{k_b} - \mathbf{k}'_a - \mathbf{k}'_b)$$

$$\times |\langle k_a k_b | T^R_{ab} | k'_a k'_b \rangle|^2 \mathrm{P}' \frac{1}{E_{ab} - E'_{ab}}$$

$$\times \{f'_a f'_b (1 \mp f_a)(1 \mp f_b) - (1 \mp f'_a)(1 \mp f'_b) f_a f_b\}. \tag{51}$$

Here, $\mathrm{P}' = \lim_{\eta \to 0} \frac{\partial}{\partial x}\frac{x}{x^2 + \eta^2}$ denotes the differentiation of the principal value and $E_{ab} = \epsilon_a + \epsilon_b$. A similar type of equation was given in [87], where the energy conserving property was pointed out. Reference [23] gave a complete treatment for classical nonideal plasmas resulting in a comparable form of collision integral.

It differs from the result of Snider [30], where a generalized kinetic equation is derived with the local hydrodynamic variable of $p^2/2m$. Here, we give a more general result including quasiparticle self-energies instead of free ones. From (49), we see that the time derivative is renormalized, if we define $\frac{\partial f_{\rm corr}}{\partial T} = \sum_b I^{\rm ret}_{ab}$. In [30], the renormalization of the time derivative leading to $\frac{\partial}{\partial T}(f + f_{\rm corr})$ is given. The derivatives with respect to space and momentum concerns, here, only the free distribution, whereas [30] it acts also on $f + f_{\rm corr}$. This difference is due to the different ways of splitting the cor-

relations into drift and collision term, as we pointed out after Eq. (14). Here, we claim to be correct in all time convolutions.

It is illustrative to compare our found f_{corr} with the kinetic equation of Baerwinkel [21,88]. He gives the renormalization of the time derivative as a Ψ functional. Whereas the momentum integral over the Ψ functional vanishes, the integral over our f_{corr} remains finite and will, therefore, contribute to the virial correction. Due to Baerwinkel, the virial correction turns out from the collision integral. The reason of this difference is again the different partition of correlations between drift and collision term. We summarized all correlations beyond Hartree-Fock into a retarded collision integral and obtain a net effect accumulated in f_{corr}. We further generalize to [21] concerning the complete time convolution of the T-matrix equation resulting in a contribution, which cancels some terms of Baerwinkels Ψ functional.

V. GENERALIZATION OF BETH-UHLENBECK CORRECTIONS TO NONEQUILIBRIUM

A. Balance equation for density

Now, we turn to the balance equation and concentrate first on the density. Thus one has to integrate the generalized kinetic equation (49) over momentum. The Boltzmann term vanishes and for the conserving total density we get from the normalization of the Wigner distribution to the free particle density $n_{\text{free}} = V_0^{-1} \sum_p f$ and the correlated density $n_{\text{corr}} = V_0^{-1} \sum_p f_{\text{corr}}$,

$$\frac{\partial}{\partial T}(n_{\text{free}}^a + n_{\text{corr}}^a)$$

$$= -\nabla_R \int \frac{d\mathbf{p}}{(2\pi\hbar)^3}(\nabla_p \Sigma_{\text{HF}}^a(pRT)) f_W^a(pRT). \quad (52)$$

We recognize that in the equilibrium limit, where no gradients occur, the sum of free and correlated density is conserved. This justifies the introduced interpretation of n_{corr}.

The correlated density takes the explicit form

$$n_{\text{corr}}^a = \frac{(2\pi\hbar)^3}{V_0\hbar} \frac{1}{V_0^4} \sum_b \sum_{k_a,k_b,k_a',k_b'} \mathrm{P}' \frac{1}{E_{ab} - E_{ab}'}$$

$$\times \{f_a' f_b'(1 \mp f_a)(1 \mp f_b) - (1 \mp f_a')(1 \mp f_b')f_a f_b\}$$

$$\times \langle k_a k_b|T(E_{ab}')|k_a' k_b'\rangle\langle k_a' k_b'|T_{\text{ex}}^+(E_{ab}')|k_a k_b\rangle, \quad (53)$$

where we introduced the summation over momentum instead of the integration and used the (in-medium) T-matrix operator in analogy to the scattering theory $\langle k_1 k_2|T|k_1' k_2'\rangle = (2\pi\hbar)^3\delta(k_1 + k_2 - k_1' - k_2')\langle k_1 k_2|T_{ab}^R(pR\tau T)|k_1' k_2'\rangle$. Now we define the two-particle *nonequilibrium* Bose-like Wigner distribution function g by

$$f_a f_b = g_{a+b}(E_{ab})(1 \mp f_a \mp f_b),$$

which is adopted from the equilibrium relations. Further, we use the free nonequilibrium retarded two-particle

Green function, which can be derived from (34)

$$G_2^{0R}(a,b,\omega) = -\frac{1 \mp f_a(pRT) \mp f_b(pRT)}{\epsilon_a(pRT) + \epsilon_b(pRT) - \omega - i\epsilon} \quad (54)$$

and obtain by introducing a dummy ω integration

$$n_{\text{corr}}^a = \frac{(2\pi\hbar)^3}{V_0\hbar} \frac{1}{V_0^4} \sum_b \sum_{k_a,k_b,k_a',k_b'} \int \frac{d\omega}{\pi}[g_{ab}(\omega) - g_{ab}(E_{ab})]$$

$$\times \mathrm{Re}G_2^{0R'}(a,b,\omega)\langle k_a k_b|T(\omega)|k_a' k_b'\rangle\mathrm{Im}G_2^{0R}(a',b',\omega)$$

$$\times \langle k_a' k_b'|T_{\text{ex}}^+(\omega)|k_a k_b\rangle. \quad (55)$$

The structure of this expression becomes more transparent if one write the operator notation omitting the momentum representation (see Appendix C),

$$n_{\text{corr}}^a = \frac{(2\pi\hbar)^3}{V_0\hbar} \sum_b \mathrm{Tr}\left\{\int \frac{d\omega}{\pi}[g_{ab}(\omega) - g_{ab}(E_{ab})]\right.$$

$$\left. \times \mathrm{Re}G_2^{0'}(a,b,\omega)T(\omega)\mathrm{Im}G_2^0(a',b',\omega)T_{\text{ex}}^+(\omega)\right\}. \quad (56)$$

Now one can apply the optical theorem derived in Appendix C to derive, finally,

$$n_{\text{corr}}^a = \frac{(2\pi\hbar)^3}{V_0\hbar}\mathrm{Tr}\sum_b\left\{\int \frac{d\omega}{\pi}[g_{ab}(\omega) - g_{ab}(E_{ab})]\right.$$

$$\left. \times \mathrm{Re}G_2^{0'}(a,b,\omega)\mathrm{Im}T_{\text{ex}}(\omega)\right\}. \quad (57)$$

This expression is just the same as derived from the expansion of the spectral function (25) leading to the Beth-Uhlenbeck approach, as we review shortly in the next section.

B. Correlated density from the expansion of the spectral function

Besides the balance equation derived from the kinetic equation, we can calculate the correlated density directly. Therefore, we turn to the evaluation of the correlated density (27), which was obtained by the expansion of the spectral function (25). The required imaginary part of the self-energy can be rewritten schematically,

$$\mathrm{Im}\Sigma(tt') = \mp i\left[G^<(t't)\mathrm{Im}T^R(tt') - \mathrm{Im}G^R(t't)T^<(tt')\right]. \quad (58)$$

The further evaluation can be performed neglecting time retardations, because we like to concentrate here on the effect of the expansion of the spectral function (25). Therefore, the frequency domain is the favorable representation. Introducing the same nonequilibrium Bose-like distribution function g like in the proceeding section, we get with (54) from (37),

$$\langle k|T^<(pR\omega T)|k\rangle = 2ig_{ab}(\omega, RT)\mathrm{Im}\langle k|T^R(pR\omega T)|k\rangle. \quad (59)$$

Introducing this equality into (58), the self-energy reads [19]

$$\mathrm{Im}\Sigma_a(pR\omega T) = -\frac{1}{V_0}\sum_{p'b}\int\frac{d\omega'}{2\pi}\mathrm{Im}\left\langle\frac{p-p'}{2}\left|T_{ab}^{\mathrm{ex}}(p+p',R,\omega',T)\right|\frac{p-p'}{2}\right\rangle[f_a(p'RT)\pm g_{ab}(\omega'RT)]$$

$$\times 2\pi\delta(\omega+\epsilon_b(p')-\omega'). \tag{60}$$

From this expression and setting $q = p + p', k = (p - p')/2$ the correlated density in (27) can be evaluated with the result

$$n_{\mathrm{corr}} = -\frac{1}{V_0^2}\sum_{bkq}\int\frac{d\omega}{2\pi}\mathrm{Im}\langle k|T_{ab}(q,R,\omega',T)|k\rangle\,[g(\omega'RT) - g_{ab}(\epsilon_a(k+q/2,RT) + \epsilon_b(k-q/2,RT))]$$

$$\times[1\mp f_a(k+q/2RT)\mp f_b(k-q/2RT)]\frac{\partial}{\partial\omega'}\mathrm{P}\frac{1}{\epsilon_a(k+q/2RT)+\epsilon_b(k-q/2,RT)-\omega'}. \tag{61}$$

Using once more (54), we obtain just the expression (57) of the last section.

Introducing the notation

$$T(kqk\omega') = \langle k|T_{ab}^{\mathrm{ex}}(q,R,\omega',T)|k\rangle$$

this expression agrees with the form [Eq. (2.18)] given in [19]. In equilibrium, it yields just the mass action law and the bound state parts as well as the scattering parts to the equation of state. Therefore we have shown, that up to the first order expansion in the retardation effects in the kinetic equation and the Beth-Uhlenbeck expression of the correlation density are equivalent.

In other words, it turns out that the memory effects describe the correlation in the system. Further, this correlations can be divided into bound state and scattering contributions according to [19,17] as will be demonstrated in the next section.

C. Bound states and scattering contributions

In order to evaluate the expression (57), which was derived now by two different ways, we introduce a further dummy ω integration and get, with the help of the spectral representation of the T matrix,

$$n_{\mathrm{corr}} = \frac{(2\pi\hbar)^3}{V_0\hbar}\sum_b\mathrm{Tr}\int\frac{d\omega}{\pi}g_{ab}(\omega)\{T_i(\omega)G_r'(\omega)$$

$$-[T_r(\omega)]'G_i(\omega)\}. \tag{62}$$

Here, we used the T operator of Appendix C. It is to be noted that all quantities are dependent of the total center-mass coordinate R and the macroscopic time t according to (38). By this way, we generalize the treatment of [19,17] to inhomogeneous nonequilibrium situations. Applying now the relations of (C4), we get, finally,

$$n_{\mathrm{corr}}^a = -\frac{(2\pi\hbar)^3}{V_0\hbar}\sum_b\int\frac{d\omega}{\pi}g_{ab}(\omega)$$

$$\times\mathrm{Tr}\{G_i(\omega)[T(\omega)G_i(\omega)[T(\omega)]^{+'}\}. \tag{63}$$

Now one can distinguish between the bound state contributions as poles in the T operator at real energies and scattering contributions [89]. The further treatment can be performed in the same way as given in [89,19,17]. The

result is

$$n_{\mathrm{corr}}^a = \frac{(2\pi\hbar)^3}{V_0^2\hbar}\sum_{bq}\left[2\sum_n g_{ab}(E_n(q))\right.$$

$$+\pi^2\sum_l\int_{k_{\mathrm{min}}}^\infty dk\,g_{ab}(\epsilon_a(q+k/2,RT)$$

$$+\epsilon_b(q-k/2,RT))\frac{1}{\pi}\frac{d\delta_l(kRT)}{dk}2\sin^2[\delta_l(kRT)]$$

$$\left.\times(2l+1)\left(1-\frac{\delta_{ab}}{2}(-1)^l\right)\right], \tag{64}$$

where the in-medium phase shifts $\delta_l(kRT)$ are introduced as in [17] in order to solve the T-matrix equation in channel l. It has to be pointed out that (64) is a generalization of the known Beth-Uhlenbeck virial correction [17] in that it requires the solution of the kinetic equation and from the resulting distribution functions the time-dependent T-matrix equation.

Similar treatments lead to virial corrections which resembles, but does not reproduce exactly, the quantum Beth-Uhlenbeck formula [22]. Later it was shown [28] that by adding a virial correction, the result of [22] cancels and the Beth-Uhlenbeck virial correction turns out. In the framework of real-time Green's-function formalism the correct second virial coefficient for nondegenerate systems was derived also in [21]. The form (64) of [89,19,17] is the most elaborate one, because it shows the partition of the correlated density into bound states and scattering states [90].

VI. VIRIAL CORRECTION TO THE ENERGY

A. Conservation of energy

In the last section, we derived the equation of state for the density. It was shown that it is equivalent to derive the correlated parts by memory effects and by the expansion of the spectral function. The same procedure can now be applied in order to derive the expression for the correlated energy. Therefore, we start from the generalized kinetic equation (49) and multiply with the quasiparticle energy ϵ_a and integrate. The correlated part can be handled in the same manner as before to end up with an expression where in (64), the function $g_{ab}(x)$ is replaced by $x/2g_{ab}(x)$. The balance equation reads

$$\frac{\partial}{\partial T}(E_{\text{free}}^a + E_{\text{corr}}^a) - \int \frac{d\mathbf{k}}{(2\pi\hbar)^3} \frac{\partial \epsilon(kRT)_a}{\partial T}[f_a(kRT) + f_{\text{corr}}^a(kRT)]$$

$$= -\nabla_R \int \frac{d\mathbf{k}}{(2\pi\hbar)^3} (\epsilon(kRT)_a \nabla_k \Sigma_a^{\text{HF}}(kRT)) f(kRT). \quad (65)$$

The same steps are carried out as were done for (64) and the correlated energy reads

$$E_{\text{corr}}^a = \frac{(2\pi\hbar)^3}{2V_o^2\hbar} \sum_{bq} \left[2\sum_n E_n(q)g_{ab}(E_n(q)) + \pi^2 \sum_l \int_{k_{min}}^\infty dk(\epsilon_a + \epsilon_b)g_{ab}(\epsilon_a(q + k/2RT) + \epsilon_b(q - k/2RT)) \right.$$

$$\left. \times \frac{1}{\pi} \frac{d\delta_l[kRT]}{dk} 2\sin^2[\delta_l(kRT)] (2l+1) \left(1 - \frac{\delta_{ab}}{2}(-1)^l \right) \right] \quad (66)$$

and $\frac{\partial f_{\text{corr}}}{\partial T} = \sum_b I_{ab}^{\text{ret}}$. This result is an extension of the known approaches to Beth-Uhlenbeck virial corrections and, to the authors knowledge, the form (66) was not given before.

Equation (65) shows that in the stationary case, not only the sum of the kinetic and mean-field energies, but also the sum of the free and correlated energies, are local hydrodynamical variables. Therefore, this is an extension of the Boltzmann theory. The correlation energy E_{corr} is just the mean interaction energy. Therefore, the found kinetic equation preserves global energy. This is the same result as may be found in [23] for binary collision approximations in classical nonideal plasmas. Instead of repeating the explicit proof, we show the thermodynamical consistency between (66) and (64) in the following.

B. Thermodynamic consistency

Up to now we have given two different virial corrections, to the energy and to the density. Between them are

the thermodynamical relations that the density is connected with the pressure P by $n = (\partial P/\partial\mu)_T$ and from the pressure one finds the energy following $(\partial U/\partial V)_T = -(\beta\partial/\partial\beta + 1)_T P$.

In order to check the thermodynamic consistency of (64) and (66) we go to the equilibrium situation, where g_{ab} is the Bose distribution function. Then we can write down the correlated density (64) as

$$n_{\text{corr}}^a = \sum_b \left\{ \frac{1}{e^{\beta(\epsilon_a + \epsilon_b - \mu_a - \mu_b)} - 1}, \Sigma \right\}. \quad (67)$$

Here, we abbreviated the sum and integrals in (64) by $\sum_b\{\}$ and give explicitly the dependence from the Bose distribution g and the self-energy Σ, which is temperature and density dependent. All other explicit functions are not important for the proof here, because no other density or temperature dependence occurs. The correlated density can now be rewritten as

$$n_{\text{corr}}^a = -\frac{1}{2\beta} \frac{\partial}{\partial\mu} \left(\frac{\sum_b\{\ln[1 - e^{-\beta(E - \mu_a - \mu_b)}], \Sigma\}}{1 - \frac{\partial}{\partial\mu}\Sigma} \right)$$

$$+ \frac{1}{2\beta} \sum_b\{\ln[1 - e^{-\beta(E - \mu_a - \mu_b)}], \Sigma\} \frac{\partial}{\partial\mu} \left(\frac{1}{1 - \frac{\partial}{\partial\mu}\Sigma} \right) + \frac{1}{2\beta} \frac{\partial}{\partial\Sigma} \sum_b\{\ln[1 - e^{-\beta(E - \mu_a - \mu_b)}], \Sigma\} \frac{\frac{\partial}{\partial\mu}\Sigma}{1 - \frac{\partial}{\partial\mu}\Sigma}. \quad (68)$$

Due to the thermodynamic relation $n = (\partial P/\partial\mu)_T$, we deduce the correlated part of the pressure. Further, we find from the thermodynamic relation $(\partial U/\partial V)_T = -(\beta\partial/\partial\beta + 1)_T P$ the correlated energy. Employing the equality valid in equilibrium,

$$\beta \frac{\partial}{\partial\beta} f_a = -(\epsilon_a - \mu) \frac{\partial}{\partial\mu} f_a,$$

and corresponding relations for Σ, we obtain by this way just the correlated self-energy (66),

$$E_{\text{corr}} = \frac{1}{2} \left\{ \frac{(\epsilon_a + \epsilon_b - \mu_a - \mu_b)}{e^{\beta(\epsilon_a + \epsilon_b - \mu_a - \mu_b)} - 1}, \Sigma \right\}_b.$$

This shows that the expressions we have derived for the correlated density and the correlated energy in the generalized Beth-Uhlenbeck approach are thermodynamically consistent.

VII. CONCLUSION AND OUTLOOK

In this paper, we derive a generalized quantum kinetic equation for dense quantum fluids employing the

real-time Green-function technique. Within the proposed time-dependent T-matrix approximation for strongly interacting Fermi or Bose systems the resulting kinetic equation is non-Markovian. This is an inevitable result of a reduction of the multiparticle dynamics to a one-body kinetic equation. The generalized ansatz used, which ensures causality, provides a retarded Bethe Goldstone equation, valid in nonequilibrium situations at all time scales. This is due to the explicit treatment of the complete time convolution of macroscopic and microscopic time scales.

The generalized kinetic equation found differs from the result of Snider [30], where a generalized kinetic equation is derived with the local hydrodynamic variable of $p^2/2m$. Here, we give a more general result including quasiparticle self-energies instead of free ones. Further differences are due to the different ways of splitting the correlations into drift and collision terms. Here, we claim to be correct in all time convolutions.

In the kinetic equation of Baerwinkel [21,88], he gives the renormalization of the time derivative as a Ψ functional. Whereas the momentum integral over the Ψ functional vanishes, the integral over our $f_{\rm corr}$ remains finite and will therefore contribute to the virial correction. Due to Baerwinkel, the virial correction turns out from the collision integral. The reason for this difference is again the different partition of correlations between drift and collision terms. We summarized all correlations beyond Hartree-Fock into a retarded collision integral and obtain a net effect accumulated in a retardation. A further generalization to [21] is that we consider the complete time convolution of the T-matrix equation resulting in a contribution, which cancels some terms of Baerwinkels Ψ functional. From the general retarded transport equation we derive an expression of virial correction, which can be identified in equilibrium with the quantum Beth-Uhlenbeck form of the density. Therefore, our derived kinetic equation is consistent with the quantum Beth-Uhlenbeck approach in equilibrium.

The correlated density can be interpreted as the density of correlated pairs containing bound states and scattering states. Moreover a correction to the energy is given, which is thermodynamically consistent with the correlated density to any approximation of quasiparticle energies. This explicit expression of correlated energy in terms of nonequilibrium phase shifts is a different result. In view of the consequent nonequilibrium treatment, we can give the generalization of the Beth-Uhlenbeck correction terms to time-dependent nonequilibrium situation of finite systems. As shown in earlier papers [17], these corrections allow a consistent treatment of the formation of deuterons and the Mott transition as a pressure ionization. Here, this results can be transmitted to transport equations on the same level of approximations. The remaining task is to solve the derived general kinetic equation. From the resulting time and space dependent distribution functions, one has to solve the time-dependent T-matrix equation. This provides the virial corrections in nonequilibrium for finite systems.

In this paper, it is shown how higher order correlations can be incorporated. Especially the question of initial correlations can be investigated. Some steps to solve this problem were made in [6,52] and citations therein. We will treat this appropriate choice of initial correlations in a forthcoming paper.

To summarize, we have shown the equivalence of the statistical description of correlated states, which are a sum of bound and scattering states, and the retarded non-Markovian transport equations. Both alternative descriptions can therefore be used to describe dense interacting Fermi or Bose systems. The presented generalized kinetic equation describes the influence of correlated two-particle states, which can be either bound or scattering contributions, to the one-particle distribution function. We do not describe the explicit formation of bound states in a kinetic equation by reaction terms. This is to be done on the three- or higher-particle level [91,40,24]. Here, we demonstrate that the inclusion of memory effects leads to a consideration of two-particle correlations already on the one-particle level.

As an open problem it remains to show how higher order corrections in the ansatz will act on the kinetic equations. This will be treated at another place. Furthermore, the direct comparison with the density matrix approach to nonequilibrium [41,44,92] will be a subject of future work.

ACKNOWLEDGMENTS

APPENDIX A: THE LANGRETH-WILKINS RULES

With the help of (9) it is possible to establish useful algebraic rules known as Langreth-Wilkins rules, which describe the way to get correlation or retarded functions from causal ones. They were first found by Langreth and Wilkins [93]. If we have, e.g., products of Green's functions and integration over inner variables,

$$C(1,2) = \int d\bar{1}\, A(1,\bar{1}) B(\bar{1},2), \tag{A1}$$

we can show that with the help of (II A) the following rules can be obtained building up correlation functions from (A1),

$$C^{\lessgtr}(1,2) = \int d\bar{1}\, A^r B^{\lessgtr} + \int d\bar{1}\, A^{\lessgtr} B^a, \tag{A2}$$

$$C^{r/a}(1,2) = \int d\bar{1}\, A^{r/a} B^{r/a}. \tag{A3}$$

Using these relations, one can find the following generalized Kadanoff-Baym equations from (12) by using the inverse interaction free Green's-function G_0^{-1} [62],

$$[G_0^{-1} - {\rm Re}\Sigma, G^{\lessgtr}] = [\Sigma^{\lessgtr}, {\rm Re}G] + \frac{1}{2}\left\{G^<, \Sigma^>\right\}$$
$$- \frac{1}{2}\left\{G^>, \Sigma^<\right\}. \tag{A4}$$

Here, $[\,,]$ and $\{\,,\}$ denote the commutator and anti-commutator of integrals (A1), respectively. This result is, in principle, equivalent to both equations (13) and is a compactly written form. Nevertheless sometimes it is of advantage to use this operator notation in order to investigate properties beyond the quasiparticle picture [42,8]. From this operator equation, it is possible to derive the time diagonal equation (14) as well [62,64].

APPENDIX B: DERIVATION OF THE GENERALIZED ANSATZ

Following Lipavsky et al. [65], we give the derivation presented in [64] and define auxiliary functions $G^{\lessgtr}_{r/a}$ by

$$G^{\gtrless}_r(t_1 t_2) = \Theta(t_1 - t_2)G^{\gtrless}(t_1 t_2),$$

$$G^{\gtrless}_a(t_1 t_2) = \Theta(t_2 - t_1)G^{\gtrless}(t_1 t_2), \qquad (B1)$$

which is just the thermodynamical Green's function originally introduced by Zubarew [92]. Applying the equation of motion to these functions, we derive interesting relations. It can be found for the time derivative of the first equation,

$$i\frac{\partial}{\partial t_1}G^<_r(t_1 t_2) = i\delta(t_1 - t_2)G^<(t_1 t_2)$$

$$+\Theta(t_1 - t_2)i\frac{\partial}{\partial t_1}G^<(t_1 t_2). \qquad (B2)$$

Replacing the time derivative on the left with the help of the first of the Kadanoff-Baym equation (13) and using the equality

$$\Theta(t_1 - t_2)\int_{-\infty}^{t_1}(\Sigma^> - \Sigma^<)G^<$$

$$= \Theta(t_1 - t_2)\int_{-\infty}^{t_2}(\Sigma^> - \Sigma^<)G^<$$

$$+\Theta(t_1 - t_2)\int_{-t_2}^{t_1}(\Sigma^> - \Sigma^<)G^<$$

$$= \Theta(t_1 - t_2)\int_{-\infty}^{t_2}(\Sigma^> - \Sigma^<)G^< + \int_{-\infty}^{\infty}\Sigma^r G^<_r, \qquad (B3)$$

we obtain for (B2) the relation

$$(G^r)^{-1}G^<_r = i\delta(t_1 - t_2)G^<(t_1 t_2) + \Theta(t_1 - t_2)$$

$$\times \int_{-\infty}^{t_2}d\bar{t}[\Sigma^r(t_1\bar{t})G^<(\bar{t}t_2)$$

$$+\Sigma^<(t_1\bar{t})G^a(\bar{t}t_2)].$$

Finally, operating with G^r from the left, we arrive at

$$G^<_r(t_1 t_2) = \Theta(t_1 - t_2)G^<(t_1 t_2)$$

$$= \mp G^r(t_1 t_2)f_W(t_2)$$

$$+ \int_{t_2}^{t_1}d\bar{t}'\int_{-\infty}^{t_2}d\bar{t}G^r(t_1\bar{t}')[\Sigma^r(\bar{t}'\bar{t})G^<(\bar{t}t_2)$$

$$+\Sigma^<(\bar{t}'\bar{t})G^a(\bar{t}t_2)], \qquad (B4)$$

where f_W is the Wigner distribution function. The same analysis can be repeated for $G^<_a$ with the result

$$G^<_a(t_1 t_2) = \Theta(t_2 - t_1)G^<(t_1 t_2)$$

$$= \pm f_W(t_1)G^a(t_1 t_2)$$

$$+ \int_{t_2}^{t_1}d\bar{t}'\int_{-\infty}^{t_1}d\bar{t}[G^<(t_1\bar{t})\Sigma^a(\bar{t}t')$$

$$-G^r(t_1\bar{t})\Sigma^<(\bar{t}'\bar{t})]G^a(\bar{t}'t_2). \qquad (B5)$$

These two equations allow, in principle, an iterative construction of $G^<(t_1 t_2)$ from its time-diagonal component, i.e., the Wigner function f. The first term gives the modified ansatz, while the iterative corrections represent an expansion in terms of the various relaxation times in the system [64].

Using only the first terms, we can write in Wigner coordinates

$$G^<(p, T, \tau) = \pm\left[f_W\left(p, T + \frac{\tau}{2}\right)G^a(p, T, \tau) - f_W\left(p, T - \frac{\tau}{2}\right)G^r(p, T, \tau)\right]$$

$$= \pm f_W\left(p, T - \frac{|\tau|}{2}\right)[G^a(p, T, \tau) - G^r(p, T, \tau)]$$

$$= \mp i f_W\left(p, T - \frac{|\tau|}{2}\right)a(p, \tau, T), \qquad (B6)$$

and analogously

$$G^>(k, T, \tau) = i\left[1 \mp f_W\left(k, T - \frac{|\tau|}{2}\right)\right]a(k, \tau, T). \quad (B7)$$

Here, a is the general spectral function which can be beyond the quasiparticle approximation.

APPENDIX C: OPTICAL THEOREMS AND DERIVATIVES

In this appendix, we give the derivation of frequently used optical theorems and related relations [17,19]. The retarded T matrix is related to the T operator of scat-

tering theory following

$$\langle k_a k_b | T^R(\omega) | k'_a k'_b \rangle = \langle k_a k_b | T | k'_a k'_b \rangle.$$

We assume an operator notation and decompose the T operator in real and imaginary part by $T = T_r + iT_i$. Then, from the identity $T = T^+(T^+)^{-1}T$, it follows

$$T_r + iT_i = T^+(T_r^{-1} - iT_i^{-1})T. \tag{C1}$$

In analogy it follows from $T^+ = T^+T^{-1}T$,

$$T_r - iT_i = T^+(T_r^{-1} + iT_i^{-1})T. \tag{C2}$$

By adding and subtracting both equations, one has

$$T_r = T^+T_r^{-1}T,$$

$$T_i = -T^+T_i^{-1}T. \tag{C3}$$

With the help of the defining T-matrix equation, $T^{-1} = V^{-1} - G$, with G denoting the two-particle Green's func-

tion, one derives the off-shell optical theorem,

$$T_i = T^+G_iT = TG_iT^+. \tag{C4}$$

In a second step, we give the derivative optical theorems with respect to the frequency derivation $T' = \partial T/\partial \omega$. Differentiating Eq. (C3), one has

$$\begin{aligned}T'_r &= T^{+'}T_r^{-1}T + T^+T_r^{-1}T' + T^+T_r^{-1'}T \\ &= T^{+'} + T' + iT^{+'}G_iT - iT^+G'_T - T^+G'_rT\end{aligned} \tag{C5}$$

and, finally,

$$T'_r = -iT^{+'}G_iT + iT^+G_iT' + T^+G'_rT. \tag{C6}$$

From the derivative optical theorem (C6), one can derive useful relations considering the trace of (62). From (C6), one gets by multiplying from the right with G_i and using the optical theorem (C4),

$$\mathrm{Tr}(T'_rG_i - T_iG'_r) = 2\mathrm{Im}\,\mathrm{Tr}(G_iTG_iT^{+'}). \tag{C7}$$

[1] G. F. Burgio, P. Chomaz, and J. Randrup, Phys. Rev. Lett. **69**, 885 (1992).
[2] G. Peilert et al., Phys. Rev. C **39**, 1402 (1989).
[3] B. ter Haar and R. Malfliet, Phys. Lett. B **172**, 10 (1986).
[4] G. Bertsch, H. Kruse, and S. D. Gupta, Phys. Rev. C **29**, 673 (1984).
[5] G. Bertsch and S. das Gupta, Phys. Rep. **160**, 189 (1988).
[6] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984).
[7] C. Greiner, K. Wagner, and P. G. Reinhard, Phys. Rev. C **49**, 1693 (1994).
[8] K. Morawetz, R. Walke, and G. Roepke, Phys. Lett. A **190**, 96 (1994).
[9] W. Bauer, Nucl. Phys. A **538**, 83 (1992).
[10] G. F. Bertsch and S. D. Gupta, Phys. Rep. **160**, 189 (1988).
[11] J. Aichelin and G. F. Bertsch, Phys. Rev. C **31**, 1730 (1985).
[12] C. Gregoire et al., Nucl. Phys. A **465**, 317 (1987).
[13] D. Klakow, G. Welke, and W. Bauer, Phys. Rev C **48**, 1982 (1993).
[14] V. Koch and C. F. Bertsch, Nucl. Phys. A **552**, 591 (1993).
[15] A. Bonasera et al., Phys. Lett. B **244**, 171 (1990).
[16] G. Wolf, W. Cassing, and U. Mosel, Nucl. Phys. A **552**, 549 (1993).
[17] M. Schmidt, G. Röpke, and H. Schulz, Ann. Phys. (N.Y.) **202**, 57 (1990).
[18] G. E. Beth and E. Uhlenbeck, Physica **4**, 915 (1937).
[19] R. Zimmermann and H. Stolz, Phys. Status Solidi B **131**, 151 (1985).
[20] L. Waldmann, Z. Naturforsch. Teil A **15**, 19 (1960).
[21] K. Baerwinkel, Z. Naturforsch. Teil A **24**, 38 (1969).
[22] M. W. Thomas and R. F. Snider, J. Stat. Phys. **2**, 61 (1970).
[23] Y. L. Klimontovich, Kinetic Theory of Nonideal Gases and Plasmas (Nauka, Moscow, 1975).
[24] Y. L. Klimontovich, D. Kremp, and W. D. Kraeft, Advances in Chemical Physics (Wiley, New York, 1987).
[25] D. Kremp, M. Schlanges, and T. Bornath, in Proceedings

of the ZIE School on Kinetic Equations and Nonlinear Optics in Semiconductors, edited by H. Stolz (Zentralinstitut für Elektronenphysik, Berlin, 1986), p. 33.
[26] G. Röpke, Phys. Rev. A **38**, 3001 (1988).
[27] P. J. Nacher, G. Tastevin, and F. Laloe, Ann. Phys. (Leipzig) **48**, 149 (1991).
[28] J. C. Rainwater and R. F. Snider, J. Chem. Phys. **65**, 4958 (1976).
[29] F. Laloe and W. J. Mullin, J. Stat. Phys. **59**, 725 (1990).
[30] R. F. Snider, J. Stat. Phys. **61**, 443 (1990).
[31] R. F. Snider, J. Stat. Phys. **63**, 707 (1991).
[32] S. Ayik, E. Suraud, M. Belkacem, and D. Boilley, Nucl. Phys. A **545**, 35 (1992).
[33] J. Randrup and B. Remaud, Nucl. Phys. A **514**, 339 (1990).
[34] J. H. Irving and R. W. Zwanzig, J. Chem. Phys. **19**, 1173 (1951).
[35] R. Balescu, Equilibrium and Nonequilibrium Statistically Mechanics (MIR, Moskau, 1978), Vol. 2.
[36] N. N. Bogoljubov, J. Phys. (USSR) **10**, 256 (1946).
[37] M. Born and H. S. Green, Proc. R. Soc. London, Ser. A **188**, 10 (1946).
[38] J. G. Kierkwood, J. Chem. Phys. **14**, 180 (1946).
[39] Y. L. Klimontovich and D. Kremp, Physica A **109**, 517 (1981).
[40] J. A. McLennan, J. Stat. Phys. **28**, 521 (1982).
[41] G. Röpke and H. Schulz, Nucl. Phys. A **477**, 472 (1988).
[42] W. Botermans and R. Malfliet, Phys. Rep. **198**, 115 (1990).
[43] E. G. D. Cohen, Physica A **194**, 229 (1993).
[44] D. N. Zubarev, V. G. Morozov, I. P. Omelyan, and M. V. Tokarchuk, Teor. Mat. Fiz. **87**, 113 (1991).
[45] T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955).
[46] P. C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).
[47] L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1964).
[48] J. Schwinger, J. Math. Phys. **2**, 407 (1961).
[49] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964).

[50] R. A. Craig, J. Math. Phys. **9**, 605 (1968).

[51] D. Kremp, M. Schlanges, and T. Bornath, J. Stat. Phys **41**, 661 (1985).

[52] P. Danielewicz, Ann. Phys. (N.Y.) **197**, 154 (1990).

[53] N. P. Landsman and C. G. van Weert, Phys. Rep. **145**, 141 (1987).

[54] W. Botermans and R. Malfliet, Phys. Lett. B **215**, 617 (1988).

[55] M. Tohyama, Phys. Rev. C **86**, 187 (1987).

[56] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 305 (1984).

[57] J. W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).

[58] B. Bezzerides and D. F. DuBois, Ann. Phys. (N.Y.) **70**, 10 (1972).

[59] J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).

[60] W. Keil, Phys. Rev. D **38**, 152 (1988).

[61] E. Calzetta and B. L. Hu, Phys. Rev. D **37**, 2878 (1988).

[62] A. P. Jauho and J. W. Wilkins, Phys. Rev. B **29**, 1919 (1984).

[63] I. B. Levinson, Zh. Eksp. Teor. Fiz. **57**, 660 (1969) [Sov. Phys. JETP **30**, 362 (1970)].

[64] A. P. Jauho, in *Quantum Transport in Semiconductors*, edited by D. Ferry and C. Jacoboni (Plenum Press, New York, 1991), Chap. 7.

[65] P. Lipavsky, V. Spicka, and B. Velicky, Phys. Rev. B **34**, 6933 (1986).

[66] K. Morawetz and D. Kremp, Phys. Lett. A **173**, 317 (1993).

[67] K. Morawetz, Ph.D. thesis, Rostock University, 1992, GSI-Report No. GSI-93-14, Maerz 1993 Darmstadt (unpublished).

[68] K. Morawetz, Phys. Rev. E **50**, 4625 (1994).

[69] G. D. Mahan, Phys. Rep. **145**, 251 (1987).

[70] W. D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Akademie Verlag, Berlin, 1986).

[71] D. Kremp, W. D. Kraeft, and A. D. J. Lambert, Physica A **127**, 72 (1984).

[72] C. Bloch and C. de Dominicis, Nucl. Phys. **10**, 509 (1959).

[73] V. M. Galitskii, Sov. Phys. JETP **7**, 104 (1958).

[74] H. A. Bethe and J. Goldstone, Proc. R. Soc. London, Ser. A **238**, 551 (1957).

[75] J. R. Barker, J. Phys. (Paris) Colloq. **42**, C245 (1981).

[76] D. Calecki and N. Pottier, J. Phys. (Paris) Colloq. **42**, C271 (1981).

[77] J. R. Barker and D. K. Ferry, Phys. Rev. Lett. **42**, 1779 (1979).

[78] A. V. Kuznetsov, Phys. Rev. B **44**, 8721 (1991).

[79] P. A. Henning, Nucl. Phys. A **546**, 653 (1992).

[80] D. B. T. Thoai and H. Haug, Z. Phys. B **91**, 199 (1993).

[81] H. Haug and C. Ell, Phys. Rev. B **46**, 2126 (1992).

[82] N. N. Bytchkova, C. W. Peletminsky, and A. A. Jatsenko, Teor. Mat. Fiz. **45**, 93 (1980).

[83] V. P. Seminozhenko, Phys. Rep. **91**, 103 (1982).

[84] K. Morawetz, Phys. Lett. A (to be published).

[85] R. Peierls, *Quantum Theory of Solids* (Oxford University Press, London, 1955).

[86] P. Lipavsky, F. S. Khan, A. Kalvova, and J. W. Wilkins, Phys. Rev. B **43**, 6650 (1991).

[87] I. Y. Goncharova, Y. A. Kukharenko, and A. S. Lalayan, Kratk. Soobshch. Fiz. **1**, 3 (1988).

[88] K. Bärwinkel, in *Proceedings of the 14th International Symposium on Rarified Gas Dynamics* (University of Tokyo Press, Tokyo, 1984).

[89] D. Kremp *et al.*, Physica A **127**, 646 (1984).

[90] T. Alm, G. Röpke, and W. Schmidt, Phys. Lett. B **301**, 170 (1993).

[91] Lowry and R. F. Snider, J. Chem. Phys. **61**, 2320 (1974).

[92] D. N. Zubarew, *Nonequilibrium Statistical Thermodynamics* (Nauka, Moscow, 1971).

[93] D. C. Langreth and J. W. Wilkins, Phys. Rev. B **6**, 3189 (1972).